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# Statistical methods applied to the study of opinion formation models: a brief overview and results of a numerical study of a model based on the social impact theory

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**Abstract**
The aim of this paper is twofold. On the one hand we present a brief overview on the application of statistical physics methods to the modelling of social phenomena focusing our attention on models for opinion formation. On the other hand, we discuss and present original results of a model for opinion formation based on the social impact theory developed by Latané. The presented model accounts for the interaction among the members of a social group under the competitive influence of a strong leader and the mass media, both supporting two different states of opinion. Extensive simulations of the model are presented, showing that they led to the observation of a rich scenery of complex behaviour including, among others, critical behaviour and phase transitions between a state of opinion dominated by the leader and another dominated by the mass media. The occurrence of interesting finite-size effects reveals that, in small communities, the opinion of the leader may prevail over that of the mass media. This observation is relevant for the understanding of social phenomena involving a finite number of individuals, in contrast to actual physical phase transitions that take place in the thermodynamic limit. Finally, we give a brief outlook of open questions and lines for future work.

## 1. Introduction

Recently, there has been growing interest among theoretical physicists in studying processes and phenomena in the fields of economy, sociology, ecology, etc, by using well-established tools already developed in the fields of statistical, condensed matter and computational physics. The application of statistical physics methods to the modelling of social phenomena has been reviewed by several authors; see e.g. [1–5]. Since the task of thoroughly quoting all recent publications in these fields is beyond the scope of this work, we have restricted ourselves to first describing the conceptual basis of the approach used and then only quoting a few relevant papers, mentioning the most interesting physical concepts associated with each of them.

A common feature of many physical systems involving a large number of interacting particles is that, in some cases, details do not matter in order to describe the collective behaviour. This observation has led to the development of the concept of universality. For example, it is broadly accepted that the Ising model is a very crude description of a real magnet or a fluid. However, close to their respective critical points the behaviour of those systems is the same. This observation is due to the fact that the existence of a diverging correlation length, which sets a natural spatial scale for the system, removes all short-range (microscopic) details and the description of the critical behaviour relies on broad-brush features of the system such as the dimensionality and range of the interactions. A simple approach to the concept of universality is achieved just by recalling that in many physical situations the collective behaviour of the elements constituting a system tends to be robust, in the sense that such a behaviour is shared by many—apparently—different systems. Considering the ubiquity of critical phenomena in the field of physics in general, and condensed matter systems in particular, their occurrence in many social phenomena too would not be surprising. In fact, many early studies of social systems focused on the behaviour of particular individuals ignoring the complexity introduced by nonlinearities in the collective behaviour. It is now largely recognized that collective behaviour cannot be described by a simple extrapolation of the individual behaviour of single elements.

One can also claim that the description of social processes by means of physically motivated models could be a naive approach that ignores the complexity of the human beings. However, it has to be recognized that in many cases—suitable to be modelled—the choices of the individuals are very restricted. So far, the more dramatic examples are those where one has only two choices, such us buying–selling, PC–Macintosh, Windows–Linux, to support/vote for a candidate (or not) in a balloted election, coffee–tea, etc. In these cases, the application of a slightly modified statistical models originally addressed to the description of two-state (or only few) spin variables has become a promising and straightforward approach.

Further discussions supporting the application of statistical models in the field of sociology can be found in references [1–5]. Furthermore, Ball has recently published an excellent overview of the historical development of the physical modelling of social processes [6]. It is worth mentioning his excellent documented argument aimed at proving that by studying the rules of collective social processes, today's statistical physicists are returning to their roots, since statistics was originated in the study of social numbers during the 17th century and, subsequently, social statistic guided Maxwell and Boltzmann to the development of the foundations of statistical mechanics [6].

For the sake of completeness, let us now acquaint the reader with a few recent and relevant developments in the statistical modelling of the collective behaviour of social groups.

The model for culture transmission proposed by Axelrod [7, 8] has recently been studied by several physicists; see e.g. [9–11]. The model is based on the fact that upon an interaction the similarity of the interacting individuals increases. A detailed study shows [9] that depending on the initial diversity there is a phase transition between an ordered monocultural state and a disordered multicultural state. Subsequent studies of this model were addressed to check the robustness of the transition in the presence of noise [11], to understand the effect of the underlying social topology [10], the influence of the dimensionality on the nature of the transition [12], etc. Recent progress in this field has recently been reviewed by San Miguel *et al* [13].

The static theory of social impact, early developed by Latané [14], and the subsequent statistical formulation of Lewenstein *et al* [15] have also been applied to the modelling of social interactions [16] and more recently as an approach to the teaching–learning process in a classroom context [17–19] where precursors of phase transitions similar to those observed in magnetic systems have been identified. Also, the same approach has been used to mimic the process of social learning via the Internet (chatting) [20].

Concerning biologically and ecologically motivated models, one should mention the study of the collective displacement of self-propelled individuals based on the model proposed by Vicseck *et al* [21] that exhibits herding behaviour and second-order (far-from-equilibrium) phase transitions. A parameter-free variant of this model has been shown to exhibit self-organized critical behaviour [22]. More recently, the occurrence of continuous phase transitions in the original model of Vicseck has been challenged by Gregory *et al* [23]. On the other hand, the role of leadership in the collective displacement of individuals has been also studied [24]. Another topic of intense activity in this field is the studies of prey–predator systems. In this case, a very rich variety of physical phenomena has been reported, including, among others, irreversible phase transitions belonging to the universality class of directed percolation [25, 26], coherent resonance [27], self-organized criticality [28], complex spatio-temporal patterns associated with altruistic behaviour of the preys [29], and oscillatory behaviour [26, 27].

Within the broad context of the statistical modelling of social behaviour, this paper is devoted to models of opinion formation. So, the aim of this paper is twofold. On the one hand, we provide a brief overview on recent progress in the study of the process of opinion formation by using physically based models (section 2), and on the other hand, we will focus on models for opinion formation based on the theory of social impact developed by Latané. This task involves the formulation of a new model and the presentation and discussion of original results. For these purposes the model will be presented and discussed (section 3) and subsequently in section 4 we will give further details of the simulation method employed in order to study the model formulated in section 3. In section 5 we will present and discuss results obtained by means of numerical simulations of that model when a social group is confronted by the competition between two different sources of opinion supported by a strong leader and the mass media. We will show that this system exhibits a very rich and complex behaviour involving far-from-equilibrium first- and second-order phase transitions and strong finite-size effects. Finally, in section 6 we will state the relevant conclusions and give a brief outlook of open questions and lines for future work.

## 2. Brief overview of social models for opinion formation

Weidlich has introduced a simple model for opinion formation [30, 31] where only two trends of opinion, say $+$ and $-$, which correspond to the existence of two political parties, were considered. The formation of an individual's opinion is due to the presence of groups of persons with the same or opposite opinion. The change in the opinion is governed by a Metropolis-like transition probability [30, 31], which also includes a social temperature and a preference parameter favouring either the $+$ or the $-$ state of opinion. This parameter plays the role of an external magnetic field as in an Ising ferromagnet. The dynamics can be derived by a standard master equation yielding unstable and stable solutions such that two nonvanishing stable symmetry breaking states exist. Subsequently, Babinec [32] has shown that by introducing periodic changes to the preference parameter, the social system exhibits stochastic resonance. It is worth mentioning that the phenomenon of stochastic resonance was introduced early to explain the 100 000-year periodicity of ice ages [33] and subsequently it has been found in various physical systems such as that a tiny periodic perturbation could become coupled to a source of noise. As stressed by Babinec [32], periodic changes in the sources of opinion are actually observed in real societies. Although these changes may be far too small in order to cause a deterministic change in the opinion of an individual, a suitable coupling to the social temperature (the noise source) induces stochastic resonance in the bistable model of Weidlich.

Recently, Kuperman and Zanette (KZ) [34] have introduced a model of opinion formation based on similar models by Weidlich [30]. In the KZ model the state of opinion of each individual is a binary variable and depends on time. The opinion of each individual can change due to three effects: (1) the interaction with the rest of the individuals, modelled by a simple majority rule, (2) the influence of fashion, modelled as the effect of some external time-varying agent (such as advertising); and (3) random changes. Small-world networks [35] are used in order to simulate the underlying social structure of the population, in agreement with typical features of social communities and relationships early discussed by Milgram [36]. The modulated 'fashion wave' coupled to the noise leads to the observation of stochastic resonance, suggesting that in the actual process of opinion formation there should be an optimal noise level for the population to respond to an external 'fashion' modulation [34].

Very recently, Tessone and Toral [37] have shown that the KZ model simulated in small-world networks [35] also exhibits system-size stochastic resonance. In this way, there is an optimal number of individuals in the social group for which the average opinion follows the fashion wave better (the second effect considered in the KZ model; see above). This result points out that the response of a social system to an external forcing agent depends in a nontrivial manner on the number of individuals, as in the case of other models for social behaviour [37].

A number of social impact models for opinion formation considered the role of social leaders and eventually their competition with the mass media and other sources of opinion [38–40]; for a review see [41]. By considering different types of mutual connections between individuals, in the noise-free limit, the models exhibit discontinuous phase transitions evidenced by rapid jumps in the majority–minority opinion proportion. The behaviour still remains in the presence of noise suggesting that sharp phase transitions could be a characteristic and persistent feature of social systems with strong (competitive) sources of opinion—leader versus mass media—in the studied case. In these cases, the individuals can change their opinion but not their spatial position [38, 39]. Subsequently, this constraint was relaxed and individuals are treated as active Brownian particles interacting via a communication field [38, 40]. This scalar field considers the spatial distribution of the individual opinions. While previous models only account for the distribution of opinion in 'social' space [39], the new version also attempts to consider the collective memory effect. In this case phase transitions have also been reported to take place between either a 'paramagnetic' phase where opposite opinions have the same probability (high-temperature and high-diffusion phase) or a 'ferromagnetic' phase with a well-defined state of opinion (low-temperature and low-diffusion phase), and a phase with spatially separated domains with a local majority of one state of opinion. This kind of model, based on the social impact theory of Latané [14], will be further discussed and generalized in section 3. It is worth mentioning that the Sznajd model [42] can be modified in order to account for the influence of advertising (i.e. the mass media) on the choice an average costumer makes between two products offered in the market [43, 44]. Depending of the level of advertising, phase-transition-like behaviour in the state of opinion has been reported [43, 44]. Furthermore, it is shown that in the limit for the size of the system going to infinity, a product with an initially low amount of customers could conquer the market with the aid of a negligibly small amount of advertising [43].

On the other hand, one of the sociological problems that has attracted attention is the building or lack of consensus. There are several models aimed at describing the dynamics of such processes in the context of opinion formation and cultural dynamics [9–11, 42, 45–58].

For example, the nonlocal model for the achievement of consensus by minority opinion spreading proposed by Galam [46, 58] shows first-order phase transitions, in the thermodynamic limit, for a well-defined initial density of minority opinion agents. However, there is a strong broadening effect of the order of $N^{-1/2}$ when finite samples of $N$-agents are simulated [58]. On the other hand, one has that in the local version of Galam's model proposed by Tessone *et al* [58] the transition from initial minority spreading to majority dominance disappears in the thermodynamic $N \rightarrow$ limit. This result points out an important difference between statistical physics problems and sociophysical ones, because the former focuses on the understanding of the behaviour of the thermodynamic limit, whereas for the latter this limit is no longer relevant in the sense that it cannot be achieved by considering social groups having a finite number of individuals. In fact, Toral and Tessone have early pointed out the relevance of the size of the social community on the behaviour of social systems and the inappropriateness of taking the thermodynamic limit in these systems; for a detailed review with careful discussions on various examples see [59].

A model for opinion formation studied by de la Lama *et al* [60] considers groups of individuals having two different opinions coexisting with an intermediate group of undecided agents (those lacking a well-defined opinion). Undecided individuals can be convinced by other agents and, due to the social temperature, agents with a given opinion can spontaneously become undecided. The existence of fluctuations in the opinion of the group as well as the observation of temporal reentrant effects indicate that one has to consider carefully the results of surveys and polls obtained even just before an electoral process. In a related context, He *et al* [61] have proposed a model for opinion formation with two competing opinions and a state of neutrality. It is found that when the two opposite opinions' fraction are equal, the neutrality fraction may affect the final opinion distribution dramatically [61].

## 3. Theoretical background on models based on the social impact theory

In this work we will discuss a model for opinion formation based on the social impact theory early proposed by Latané [14], which was later statistically formulated by Lewenstein *et al* [15].

According to Latané [14] when the interaction between individuals of a social group is considered, the impact ($I$) of an individual (the source) on another one (the target) depends on at least two factors: (i) the 'strength' or 'intensity' of the source to the target, which is determined, for example, by the source's social and economic status, age, its credibility to persuade and become supported, prior relationships with or future power over the target, etc, and (ii) the 'immediacy' that accounts for the social 'distance' between the source and the target, which is determined, for example, by social, cultural, economic and religious affinities, etc.

Since we are interested in cases where the number of individuals ($N$) may be large ($N > 2$), a careful treatment of the situation becomes necessary. In fact, as pointed out by Latané [14] if one considers the social impact of many sources on a single target the psychosocial law (i.e., the second principle of Latané's theory) states that a marginal decreasing effect on $I$ operates, and the impact is not simply proportional to $N$ but one has $I \propto N^{\alpha}$, where $\alpha \leqslant 1$ is an exponent.

On the other hand, when a single source of impact acting on many targets is considered, the division of the social impact (i.e., the third principle of Latané's theory [14]) becomes relevant and one has $I \propto N^{-\beta}$, where $\beta$ is also an exponent. This effect accounts for the fact that an individual, in the presence of many others, will feel a diminution of the impact as compared to the case that he/she would be alone.

In order to formulate a model for opinion formation we will follow the ideas already proposed by Hołyst *et al* [38–40]; for a review on this subject see [41]. However, the original model will be generalized in order to fully account for the principles of Latane's theory. We will restrict ourselves to the case that the opinion of every individual ($j = 1, 2, \ldots, N$) can assume only two states. So, in the formulation of the model this assumption is accounted for by considering a two-state 'spin' variable $\sigma_j = \pm 1$ representing the states of opinion.

On the other hand, the strength of the $j$th individual is denoted by $S_j$. Furthermore, the immediacy between the $j$th and the $i$th individuals is given by $n_{ij}$, such that in general one has $n_{ij} \neq n_{ji}$.

So, in order to evaluate the social interaction among individuals, one has to compute the social impact on the $i$th individual ($I_i$) due to the remaining members of the group, which can be written as [38, 39]

$$I_i = \sum_{j=1}^{N} S_j n_{ij} \sigma_j. \tag{1}$$

Equation (1) implies that the impact of all sources acting on the $i$th individual has the same statistical weight. However, in order to generalize the original model of Hołyst *et al* [38, 39], one has to account for the—already discussed—second principle of Latané's theory. So, we define the average impact $\langle I_i \rangle$ acting on the $i$th individual according to

$$\langle I_i \rangle = I_i / N. \tag{2}$$

Then, the impact of all $N$-interacting individuals of the group can be evaluated according to

$$I_i^G = \langle I_i \rangle N^{\alpha} = I_i N^{\alpha - 1}, \tag{3}$$

where $\alpha \leqslant 1$ is an exponent.

After considering the social interaction among the members of the group, we will focus our attention on introducing the competitive action of two additional sources of opinion: on the one hand, an external source due to the mass media, which may account for the influence of newspapers, television, radio and all sources of propaganda in general; and on the other hand, an internal source due to the presence of a strong leader within the social group.

Let us first introduce the impact of the external source $hN$ acting uniformly on all $N$ individuals and favouring one of the states of opinion. This source will subsequently be referred to as the influence of the mass media. Since the mass media are assumed to impact over many individuals the third principle of Latané's theory has to be considered. Consequently, the external source undergoes the division of its impact on the social group, so that its impact on the $i$th individual ($I_i^E$) is given by

$$I_i^E = (hN)/N^{\beta} = hN^{1-\beta}, \tag{4}$$

where $\beta$ is an exponent.

Now, in order to introduce an internal source given by a strong leader, we assume that one individual (say individual $j = 1$) does not change at all his/her opinion ($\sigma_1 = 1$) and his/her 'strength' or 'intensity' to produce an impact on the society is greater than the average. The latter condition is simply achieved by taking $S_1 \gg \langle S \rangle$, where $\langle S \rangle = \frac{1}{(N-1)} \sum_{j=2}^{N} S_j$ is the average strength of the social group. In the present work we have taken $S_1 = 10\langle S \rangle$.

Summing up, the total impact on the $i$th individual is given by the contribution of equations (3) and (4), and for the particular case $\alpha = 1$ and $\beta = 0$ one recovers the model of Hołyst *et al* [38, 39].

As pointed out by Latané [14], his original theory is essentially static: it provides laws to evaluate the impact, and the dynamic behaviour of the targets of such an impact is not addressed. This shortcoming has already been overcome by the subsequent statistical formulation of the theory [15]. In our case, the impact influences the opinion of the individuals, which changes in time according to the following dynamic rule:

$$\sigma_i(t+1) = \begin{cases} +1 & \text{with probability } \mathbf{P} \\ -1 & \text{with probability } \mathbf{1-P}, \end{cases} \tag{5}$$

where the transition probability $\mathbf{P}$ is given by

$$\mathbf{P} = \exp(X)/(\exp(-X) + \exp(X)), \tag{6}$$

where

$$X = \beta_G I_i^G + \beta_E I_i^E. \tag{7}$$

After inspecting equations (6) and (7) one can recognize a Metropolis-like transition probability [62] where the impacts play the role of the Hamiltonian[3] of standard equilibrium systems and the terms $\beta_G$ and $\beta_E$ are the so-called 'inverse social temperatures' ($\beta^* = 1/T$), which are often introduced in the transition rates given by equation (6); see e.g. [17–19, 38, 39] and references therein. The social temperature accounts for the noise (misunderstandings, lack of attention, etc) in the communications. Since, in principle, the noise due to the interaction among individuals in the social group may be different from that of the interaction between the mass media and the individuals we have introduced two terms, given by $\beta_G$ and $\beta_E$, respectively.

## 4. Details of the numerical simulations

The system under study is a cellular automaton, so all individuals are updated simultaneously during each (discrete) time step (TS). The unit of time is just TS.

Furthermore, the strength parameters of the interaction $S_j$ are random variables uniformly distributed within the interval $0 < S_j \leqslant 2\langle S\rangle$, where $\langle S\rangle$ is the average strength of the individuals, which along the simulations is taken as $\langle S\rangle = 1$.

Individuals are placed in a one-dimensional ring and the immediacy is taken as the inverse of the geometric distance on the ring ($n_{ij} = 1/d_{ij}$). On the other hand, the self-immediacy term ($n_{jj}$) describes the self-support of the individual that represents the social inertia preventing his/her change of opinion. In the simulations $n_{jj}$ is taken at random and uniformly distributed according to $0 < n_{jj} < 2\langle S_S\rangle$, where $\langle S_S\rangle$ is the average value of the self-support, which is taken as $\langle S_S\rangle = 5$.

We remind the reader that the strong leader of the social group (individual $j = 1$) does not change his/her opinion at all ($\sigma_1 = 1$) and his/her 'strength' is greater than the average; namely we have taken $S_1 = 10\langle S\rangle$.

Simulations are performed for $64 \leqslant N \leqslant 16\,384$ individuals, starting from random distributions of the opinions (average initial value $\sigma_0 = 0$). Subsequently, the dynamic evolution of the system is simulated and after a transient period a stationary state is achieved. The average opinion for the set of parameters used in the simulation is computed during this stationary regime. Averages over 50 different initial configurations are usually performed.

## 5. Results and discussion

Figure 1 shows plots of the average opinion ($\langle\sigma\rangle$), corresponding to groups of individuals of different size, versus the strength of the external source ($h$) (mass media). In all the examples

[3] In general one has that the couplings can be unsymmetric ($n_{ij} \neq n_{ji}$), and in that case, a Hamiltonian may not exist. However, the results presented below are obtained by assuming symmetric couplings where a Hamiltonian can be defined.

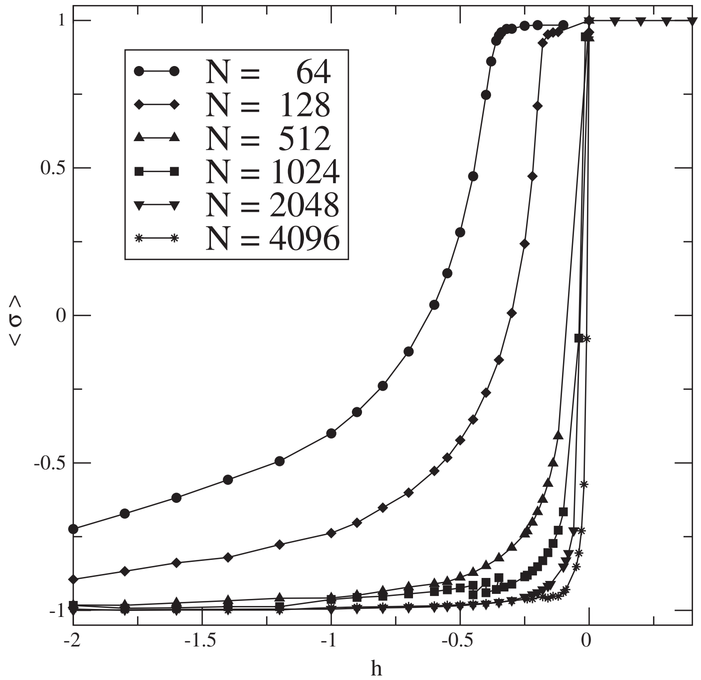


**Figure 1.** Plots of the average opinion ($\langle\sigma\rangle$) versus the strengths of the mass media ($h$). Results corresponding to groups of different size as listed in the figure, and obtained for $\alpha = 1$ and $\beta = 0$.

shown in figure 1 we have taken $\alpha = 1$ and $\beta = 0$, which corresponds to the original model for opinion formation proposed by Hołyst *et al* [38–40]. Also, for the sake of comparison we have assumed $\beta_G = \beta_E = 1$ for the 'inverse social temperatures'.

Since the leader has (unchanged) $\sigma_1 = 1$, all members of the group share that opinion for $h > 0$, in agreement with the fact that all sources of impact have the same opinion. However, for $h < 0$ one has a dynamic competition between the opinion of the leader and that of the mass media. For small groups the opinion of the leader still prevails even for negative values of $h$. In these cases one observes a smooth change in the average opinion of the group from negative to positive, which corresponds to mass media- and leader-dominated regimes, respectively. When the number of individuals in the group is increased (e.g. $N \geqslant 1024$) the change of opinion is rather sharp and resembles a first-order transition-like behaviour. Of course, this behaviour could be understood as a precursor of a true transition that will occur in the $N \to \infty$ limit only.

It is worth mentioning that shifting and rounding effects due to the finiteness of the system, which may be undesirable during the study of phase transitions of actual physical systems where the attention is focused on the $N \to \infty$ limit, are of great interest in the field of sociodynamics. In fact, in actual society population systems not only is it impossible to achieve the thermodynamic limit, but also the study of groups with a relatively reduced number of individuals may be relevant for understanding the behaviour of small communities such as students in a classroom, employees in a company, members of an association, small villages, etc. Within this context our findings are consistent with the intuitive expectation that the opinion of a strong leader of a small community may prevail over the influence of external sources. In a closely related context, another interesting effect introduced by the finiteness of the group is the occurrence of system size stochastic resonance in a model for opinion formation, as has recently been reported by Tessone *et al* [37]. Furthermore, very recently the occurrence of a noise-induced transition in finite systems has been reported for the case of Axelrod's model for the dissemination of culture in a social group [10–12].

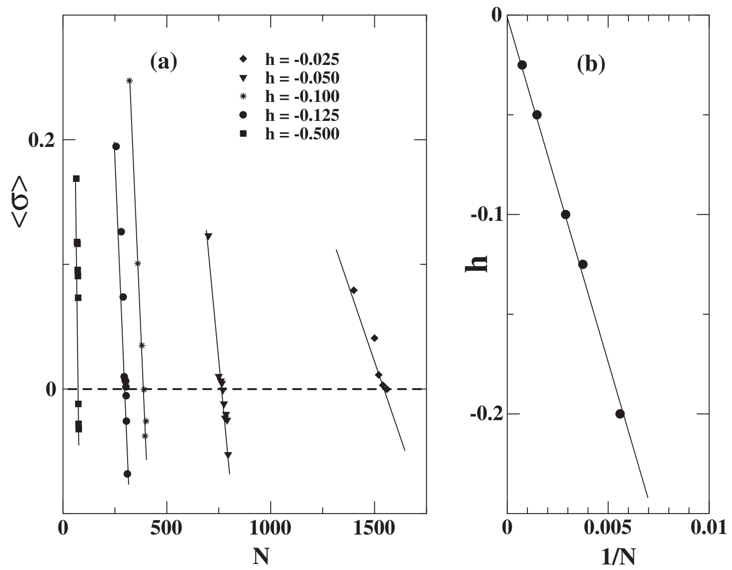


**Figure 2.** (a) Plots of the average opinion ($\langle\sigma\rangle$) versus the number of individuals of the social group $N$ as obtained for different strengths of the mass media ($h$). (b) Plot of the value of the strength of the mass media ($h$) at the transition observed in the state of opinion versus $1/N$. Results corresponding to $\alpha = 1$ and $\beta = 0$.

In order to further understand the size dependence of the change of the average opinion of the group, from the mass media to the leader dominated regimes, within the framework of the original model of Hołyst *et al* [38, 39], we have performed detailed simulations close to $\langle\sigma\rangle \simeq 0$ for different values of both $h$ and the number of individuals $N$, as shown in figure 2(a). As expected from the results already shown in figure 1, it is found that the strength of the mass media has to increase, when the number of individuals decreases, in order to induce the transition between the different states of opinion (dominated by the leader for $\langle\sigma\rangle > 0$ and by the mass media for $\langle\sigma\rangle < 0$). In particular, we are interested in the number of individuals ($N^*$) that is necessary in order to produce a change in the state of opinion for a given value of $h$. By using the results shown in figure 2(a), we found a dependence given by $h \propto 1/N^*$ (figure 2(b)). Figure 2(b) also shows that for $N^* \to \infty$, the transition actually takes place for $h = 0$. In this case one has a true first-order transition in the state of opinion.

Since mean-field calculations are mostly restricted to the low- (social) temperature limit, we have taken advantage of the numerical simulation method in order to further investigate the role of the noise, by keeping $\alpha = 1$ and $\beta = 0$, which corresponds to the model proposed by Hołyst *et al* [38–40]. Also, we have assumed $\beta_G = \beta_E$ for the 'inverse social temperatures' for the sake of simplicity. Figure 3(a) shows results corresponding to $N = 2048$, indicating that the low-temperature approach holds even for social temperatures as larger as $T = 100$. In fact, the difference with the curves obtained at lower temperatures, such as $T = 1$ and 2, are almost negligible. This result strongly suggest that our previous conclusions, concerning the role of finite-size effects on the transition, remain valid for a wide range of social temperatures. Of course, by further increasing $T$ one observes shifting and rounding effects, e.g. for $T = 1000$ in figure 3(a). On the other hand, by keeping the temperature constant ($T = 100$, in figure 3(b)) and increasing the number of individuals, we observe that the first-order nature of the transition can be expected to remain valid in the thermodynamic limit.

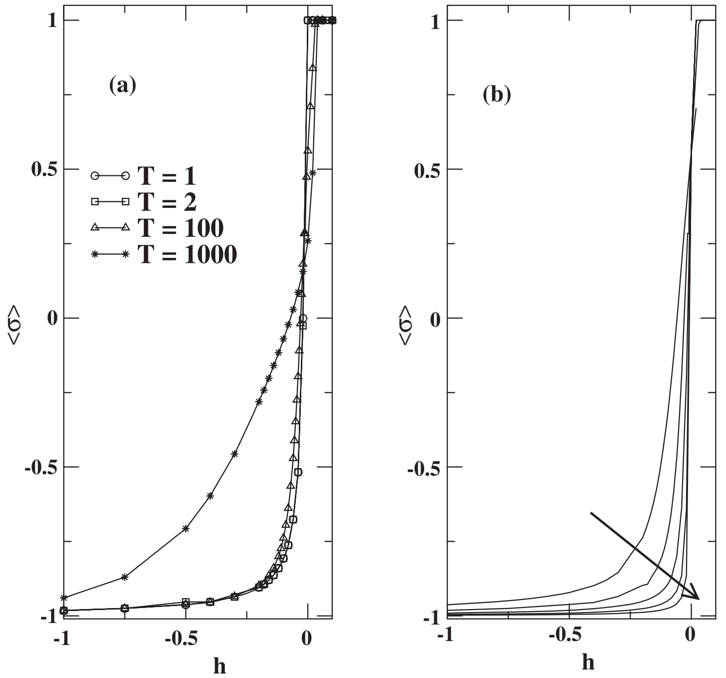


**Figure 3.** Plots of the average opinion ($\langle\sigma\rangle$) versus the strength of the mass media ($h$). Results corresponding to $\alpha = 1$ and $\beta = 0$. (a) Data obtained for $N = 2048$ and different social temperatures, as listed in the figure. (b) Results corresponding to $T = 100$ and five different values of the number of individuals $N = 1024, 2048, 4096, 8192$ and 16 384. The arrow at the low-right corner shows the trend obtained by increasing $N$.

Let us now discuss the results obtained by using the generalized model proposed in this work. For this purpose we have selected $\beta_G = \beta_E = 1$ for the 'inverse social temperatures' not only for the sake of comparison with the original model for opinion formation proposed by Hołyst *et al* [38, 39], but also considering that the low-temperature behaviour is quite robust, and consequently the results would have a wider range of validity.

In order to study the effect of the exponent $\alpha$ we have performed simulations by keeping $\beta = 0.5$ constant and varying $\alpha$, as shown in figure 4. In spite of the fact that for $\alpha < 1$ the impact of the social group and its interaction with the leader becomes weakened, the first-order transition-like behaviour remains robust when $\alpha$ is changed (figure 4). Also, when the impact of the social interaction between members of the group is strong (e.g. for $\alpha \geqslant 0.75$ in figure 4) the opinion of the leader still prevails over that of the mass media for slightly negative values of $h$. This observation is consistent with the fact that $\beta = 0.5$ represents a rather strong division of the impact of the mass media that becomes considerably weakened. The effect of such an impact division is somewhat compensated when the social interactions among the individuals become weakened ($\alpha \leqslant 0.5$ in figure 4).

Figure 5 shows plots of the average opinion $\langle\sigma\rangle$ versus the strength of the mass media ($h$) obtained by keeping $\alpha = 0.5$ constant and varying the value of $\beta$. Results are obtained by using samples of $N = 2048$ individuals. Let us recall that according to the second principle of Latané's theory [14], $\alpha = 0.5$ implies a rather important saturation effect on the social impact due to both the interaction among the individuals of the social group and the influence of the leader. For $\beta = 0$ the mass media do not undergo a division of the impact, and a sharp

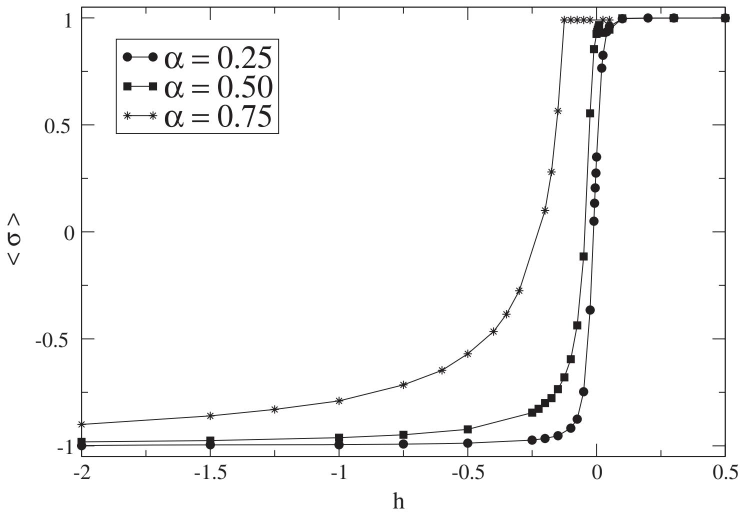


**Figure 4.** Plots of the average opinion ($\langle\sigma\rangle$) versus the strength of the mass media ($h$). Data obtained for $\beta = 0.5$ (constant) and varying $\alpha$ as listed in the figure.

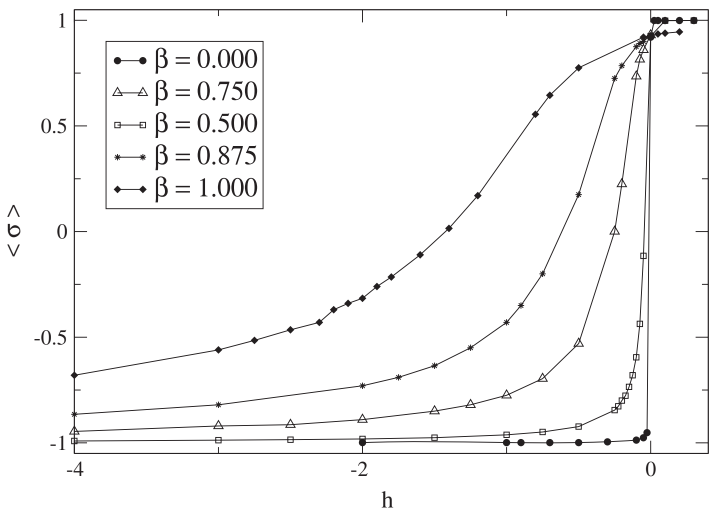


**Figure 5.** Plots of the average opinion ($\langle\sigma\rangle$) versus the strength of the mass media ($h$). Data obtained for $\alpha = 0.5$ (constant) and varying $\beta$ as listed in the figure.

transition between both states of opinion is observed. In fact, this transition is even sharper than that observed for the case of Hołyst model with $\alpha = 1$ (see figure 1) due to the weakening of the influence of the leader, which drives the social group to the $\sigma = 1$ opinion state. For $\beta = 0.5$ one has that a first-order-like transition between opposite states of opinion is still observed (see figure 5). On the other hand, within the range $0.75 \leqslant \beta \leqslant 0.875$ one observes smooth changes in the opinion of the individuals that may resemble weak first-order-like behaviour. However, for $\beta = 1$ the division of the impact affecting the mass media is strong enough to destroy the first-order transition-like behaviour and one observes a smooth change in the state of opinion of the social group when $h$ is weak enough.

In view of our previous results (see figure 1) showing that the shape of the curves may be influenced by shifting and rounding effects induced by the finiteness of the sample, we have performed a detailed study of finite-size effects for various cases, involving different values of both $\alpha$ and $\beta$. Results obtained by taking $\alpha = 0.5$ and $\beta = 0.5$ (not shown here for the sake of space) and increasing the size of the system from $N = 2048$ up to $N = 16\,384$ shown a well-

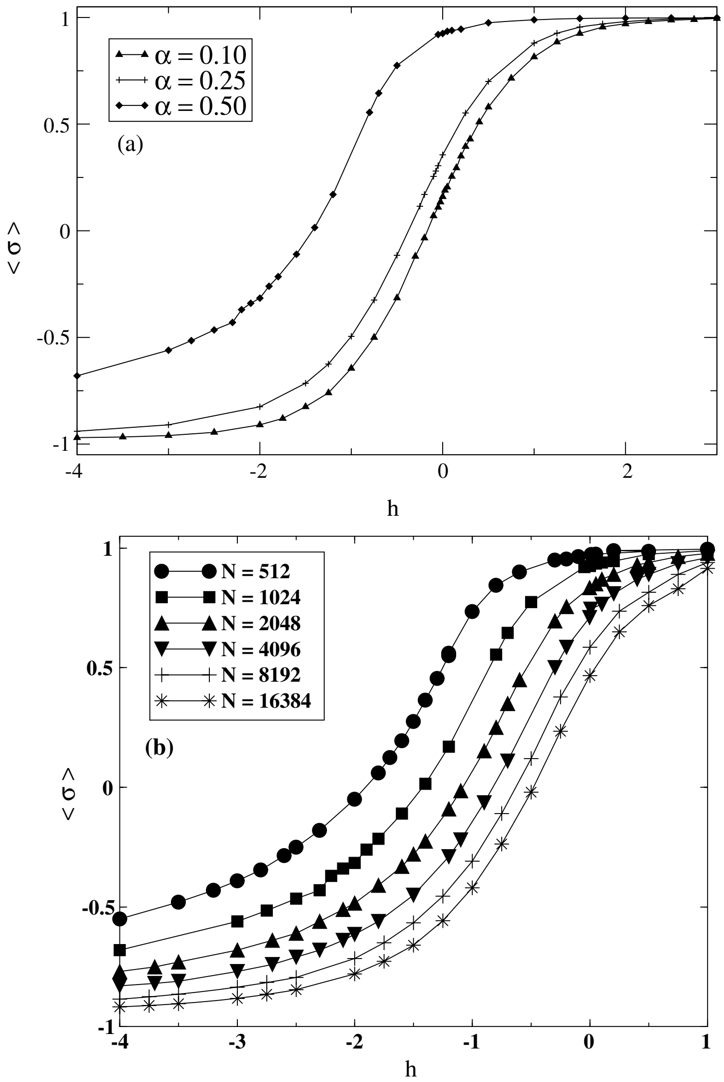


**Figure 6.** Plots of the average opinion ($\langle\sigma\rangle$) versus the strength of the mass media ($h$) as obtained by keeping $\beta = 1$ (constant). (a) Results corresponding to $N = 2048$ obtained by varying $\alpha$ as listed in the figure. (b) Data obtained by keeping $\alpha = 0.5$ (constant), for systems of different size as listed in the figure.

defined, sharp transition, for all system sizes, while finite-size effects are almost negligible. These results confirm that the robustness of the sharp transition of the original Hołyst's model (figure 1) becomes enhanced when the division of the impact and the saturation effect are rather important.

On the other hand, it is also worth analysing the case $\beta = 1$ (figure 6(a)) such that the division of the impact is maximum. In this case the contribution of mass media becomes independent of $N$ (see equation (4)) and the changes in the state of opinion are smooth. Also, when $\alpha$ is decreased one observes a monotonic shift of the curves toward larger values of $h$, which is consistent with the weakening of the social interaction of the individuals and the

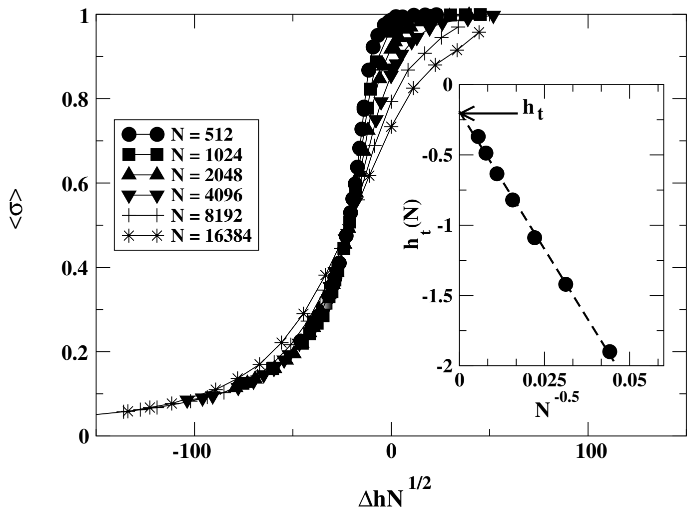


**Figure 7.** Plots of the data displayed in figure 6(b) showing the average opinion ($\langle\sigma\rangle$) versus the scaled strength of the mass media ($\Delta h N^{1/2}$), with $\Delta h = h(N) - h(\infty)$. The inset shows a plot of the $N$-dependent threshold of the mass media versus $N^{-1/2}$ (see equation (8)).

leader. In view of these findings, the size dependence of the results, for the case of the largest division of the impact of the mass media ($\beta = 1$), has been studied in further detail, as shown in figure 6(b). Here the plot corresponding to $\alpha = 0.5$ and $N = 2048$ has also been included for the sake of comparison with figures 5 and 6(a). In all cases, the first-order transition-like behaviour is no longer observed for such a strong division of the impact of the mass media.

The observation of large shifting and rounding effects, as well as the continuous nature of the change in the state of opinion of the group that follows after inspection of figure 6(b), strongly suggests the application of the finite-size scaling theory, already developed for the understanding of second-order phase transitions in the field of condensed matter physics [62]. So, considering the strength of the mass media exactly at the threshold where the opinion of the group changes between the regime dominated by the mass media ($\langle\sigma\rangle < 0$) and the regime dominated by the leader ($\langle\sigma\rangle > 0$) for a system of size $N$, namely $h_t(N)$, the following Ansatz can be formulated:

$$h_t(N) = h_t(\infty) + AN^{-\theta}, \tag{8}$$

where $h_t(\infty)$ is the threshold in the thermodynamic limit, $A$ is a constant and $\theta$ is the shifting exponent. Notice that within the framework of the finite-size scaling theory $\theta$ and the correlation length exponent $\nu$ are related according to $\theta = 1/\nu$. Fits of the data, including large groups of $N = 32\,768$ members, are consistent with $\theta \simeq 0.50 \pm 0.10$. Clear departures from the proposed Ansatz are observed for exponents outside the above range. So, for the sake of simplicity hereafter we have taken $\theta = 1/2$. The inset of figure 7 shows a plot of $h_t(N)$ versus $N^{-1/2}$, which supports the validity of the Ansatz given by equation (8) and also allows us to extrapolate the value $h_t(\infty) = 0.210 \pm 0.005$. On the other hand, figure 7 shows an attempt to collapse all the data available for different sample sizes, just by rescaling the horizontal axis. This simple plot shows an excellent collapse around the centre of the 'S'-shaped curves of the average opinion. Furthermore, this plot is also instructive because it clearly shows the asymmetry of the branches for $\langle\sigma\rangle < 0$ and $\langle\sigma\rangle > 0$. This result indicates that it is impossible to obtain a single collapsed curve describing the whole behaviour of the data. Furthermore, this finding is evidence of the different underlying mechanisms. On the one hand, one has the transition between the regime dominated by the mass media ($\langle\sigma\rangle < 0$) and the threshold $\langle\sigma\rangle = 0$. This change of opinion is induced by the influence of the leader that starts to prevail over the mass media. On the other hand, one also has the transition, driven by the mass media, between the regime dominated by the leader $\langle\sigma\rangle > 0$ and the threshold $\langle\sigma\rangle = 0$.

So, in view of the validity of equation (8) we have strong evidence of the second-order nature of the transition for the case of the largest division of the impact with $\beta = 1$. Therefore, changing the strength of the division of the impact undergone by the mass media one actually moves along a line of first-order transitions ($\beta < 1$) that become increasingly weakened, ending finally in a second-order point just for $\beta = 1$.

Finally, it is worth mentioning that the occurrence of continuous nonequilibrium transitions has recently been reported for Axelrod's model of social interaction in several complex networks [10–12]. In particular, the smooth transitions observed by using random scale-free networks are strongly shifted and rounded due to finite sample effects. In this case a careful extrapolation to the thermodynamic limit (number of individuals of the group $N \to \infty$) reveals that the transition point diverges with $N$, so that the effective transitions are only observed in finite systems [10–12]. Thus, in the thermodynamic limit the actual expected transition disappears. This result is in contrast to our finding, because in the our model for opinion formation the critical threshold converges towards a finite value for $N \to \infty$ (see inset of figure 7 for $\beta = 1$) and a finite-size scaling study reveals the existence of a true transition in the thermodynamic limit.

## 6. Conclusions

We have briefly discussed various models for opinion formation that have been studied by using well-established techniques already developed in the fields of statistical and condensed matter physics. Special attention has been focused on those models based on the social impact theory of Latané [14].

Subsequently, we have proposed and studied a generalization of the model developed by Hołyst *et al*, for opinion formation in a social group [38, 39]. Our proposal accounts for the three principles of the social impact theory of Latané [14].

The competitive social impact acting on the individuals, due to, on the one hand, a strong leader and, on the other hand, to the mass media, both supporting opposite opinions, leads to the occurrence of phase-transition-like behaviour. These transitions take place between states where the opinion of the group is either dominated by the mass media or by the leader. It is found that the influence of the leader increases when the size of the social group decreases. Also, for small groups the transition between different opinions becomes smooth, in contrast to the sharp transitions observed in large groups. These results point out the relevance of the leaders in small communities.

We have also studied the influence of the social temperature, which accounts for the noise (misunderstandings, lack of attention, etc) in the communications among individuals and the information received from the mass media. Sharp transitions are also observed for low temperatures and a study of finite-size effects reveals that this behaviour also remains in the thermodynamic limit.

Considering both saturation effects influencing the interaction among individuals, and the division of the impact that the mass media undergoes due to the fact that it is addressed to many individuals, we observe a very rich scenery due to the interplay of these effects and the size of the group. On the one hand, the first-order transition-like behaviour, exhibited by the original model of Hołyst *et al*, also prevails provided that such a division is weak enough. This behaviour even becomes enhanced and results are almost free of finite-size effects for $\alpha = 0.5$ and $\beta = 0.5$. However, in those cases where the division of the impact is large, sharp transition-like behaviour is no longer observed for small enough groups, and the social group changes its opinion smoothly when the strength of the mass media is varied. For these cases the sharp first-order-like behaviour is expected to become dominant in the thermodynamic limit. On the other hand, for $\beta = 1$ sharp transitions are no longer observed and a careful finite-size scaling study reveals the occurrence of a second-order phase transition. So, a line of first-order transitions observed for $\beta < 1$ ends in a second-order one for $\beta = 1$.

These results are in contrast to previous studies of opinion formation models exhibiting persistent and robust first-order-like transitions [38–40] and point out that opinion formation in social groups exhibits a richer scenery of complex behaviour when saturation effects and the division of the impact are properly considered. In addition, our results show that the opinion of a strong leader becomes relevant and may prevail over that of the mass media in small communities of individuals.

Finally, we would like to address a few lines of research for futures studies that may lead to interesting complex behaviour. Due to the ubiquity of first-order transitions in many sociodynamic models the study of the possible emergence of dynamical phase transitions driven by an oscillatory control parameter, such as those already observed in some magnetic model systems [63], would be of great interest for the understanding the social inertia of groups of individuals. Within this context we have already performed a preliminary study of this subject [64] and further work is under progress. This issue is closely related to the observation of stochastic resonance in various models of opinion formation [32, 37]. Another interesting topic would be the formulation and study of a model suitable to account for the emergence of leaders in an interacting society and eventually the onset of 'revolutionary' groups.

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## References

[1] Weidlich W 2002 *Sociodynamics: A Systematic Approach to Mathematical Modelling in Social Sciences* (London: Taylor and Francis)
[2] Weidlich W 1991 *Phys. Rep.* **204** 1
[3] Stauffer D, Moss de Oliveira S, de Oliveira P M C and Sa Martins J S 2006 *Biology, Sociology, Geology by Computational Physicists* (Amsterdam: Elsevier)
[4] Stauffer D 2004 *Physica* A **336** 1
[5] Galam S 2004 *Physica* A **336** 49
[6] Ball P 2002 *Physica* A **314** 1
[7] Axelrod R 1997 *J. Confict. Resol.* **41** 203
[8] Axelrod R 1997 *The Complexity of Cooperation* (Princeton: Princeton University Press)
[9] Castellano C, Marsili M and Vespignani A 2000 *Phys. Rev. Lett.* **85** 3536
Vilone D, Vespignani A and Castellano C 2002 *Eur. Phys. J.* B **30** 399
[10] Klemm K, Eguíluz V M, Toral R and San Miguel M 2003 *Phys. Rev.* E **67** 026120
[11] Klemm K, Eguíluz V M, Toral R and San Miguel M 2003 *Phys. Rev.* E **67** 045101R
[12] Klemm K, Eguíluz V M, Toral R and San Miguel M 2003 *Physica* A **327** 1
[13] San Miguel M, Eguíluz V M, Toral R and Klemm K 2005 *Comput. Sci. Eng.* **7** 67
[14] Latané B 1981 *Am. Psychol.* **36** 343
[15] Lewenstein M, Nowak A and Latané B 1992 *Phys. Rev.* A **45** 763
[16] Kohring G A 1996 *J. Physique* I **6** 301
[17] Bordogna C M and Albano E V 2001 *Phys. Rev. Lett.* **87** 118701
[18] Bordogna C M and Albano E V 2001 *Int. J. Mod. Phys.* C **12** 1241
[19] Bordogna C M and Albano E V 2003 *Physica* A **329** 281
[20] Bordogna C M and Albano E V 2002 *Eur. Phys. J.* B **25** 391
[21] Vicsek T *et al* 1995 *Phys. Rev. Lett.* **75** 1226

[22] Albano E 1996 *Phys. Rev. Lett.* **77** 2129
See also Albano E 1996 *Phys. Today* (December) 9
[23] Grégoire G and Chaté H 2004 *Phys. Rev. Lett.* **92** 025702
[24] Couzin I *et al* 2005 *Nature* **443** 513
[25] Boccara N, Robin O and Roger M 1994 *Phys. Rev.* E **50** 4531
[26] Rozenfeld A F and Albano E V 1999 *Physica* A **266** 322
[27] Rozenfeld A F, Tessone C J, Albano E V and Wio H S 2001 *Phys. Lett.* A **280** 45
[28] Rozenfeld A F and Albano E V 2004 *Phys. Lett.* A **332** 361
[29] Rozenfeld A, Gruver J L, Albano E V and Havlin S 2006 *Physica* A **369** 817
[30] Weidlich W 1991 *Phys. Rep.* **204** 1
[31] Haken H 1977 *Synergetics. An Introduction* (Berlin: Springer)
[32] Babinec P 1981 *Phys. Lett.* A **225** 179
[33] Benzi R, Sutera A and Vulpiani A 1981 *J. Phys. A: Math. Gen.* **14** L453
[34] Kuperman M and Zanette D 2002 *Eur. Phys. J.* B **26** 387
[35] Watts D J and Strogatz S H 1998 *Nature* **393** 440
[36] Milgram S 1967 *Psychol. Today* **2** 60
[37] Tessone C and Toral R 2005 *Physica* A **351** 106
[38] Hołyst J, Kacperski K and Schweitzer F 2000 *Physica* A **285** 199
[39] Kacperski K and Hołyst J 2000 *Physica* A **287** 631
[40] Schweitzer F and Hołyst J A 2000 *Eur. Phys. J.* B **15** 723
[41] Hołyst J A, Kacperski K and Schweiter F 2001 Social impact models of opinion dynamics *Annual Reviews of Computational Physics IX* ed D Stauffer (Singapore: World Scientific) p 253
[42] Sznajd-Weron K and Sznajd J 2000 *Int. J. Mod. Phys.* C **11** 1157
Sznajd-Weron K and Sznajd J 2002 *Int. J. Mod. Phys.* C **13** 115
Sznajd-Weron K 2002 *Phys. Rev.* E **66** 046131
[43] Sznajd-Weron K and Weron R 2003 *Physica* A **324** 437
[44] Sznajd-Weron K 2005 *Acta Phys. Pol.* B **36** 2537
[45] Galam S, Chopard B, Masselot A and Droz M 1998 *Eur. Phys. J.* B **4** 529
Galam S and Zucker J D 2000 *Physica* A **287** 644
[46] Galam S 2002 *Eur. Phys. J.* B **25** 403
Galam S 2003 *Physica* A **320** 571
[47] Deffuant G, Neau D, Amblard F and Weisbuch G 2000 *Adv. Complex Syst.* **3** 87
Weisbuch G, Deffuant G, Amblard F and Nadal J-P 2002 *Complexity* **7** 55
Amblard F and Deffuant G 2004 *Physica* A **343** 453
[48] Hegselmann R and Krausse U 2002 *J. Artif. Soc. Social Sim.* **5** 3
[49] Slanina F and Lavicka H 2003 *Eur. Phys. J.* B **35** 279
[50] He M, Sun Q and Wang H 2004 *Int. J. Mod. Phys.* C **15** 767
[51] Stauffer D, Souza A O and Moss de Oliveira S 2000 *Int. J. Mod. Phys.* C **11** 1239
Ochrombel R 2001 *Int. J. Mod. Phys.* C **12** 1091
Stauffer D 2002 *Int. J. Mod. Phys.* C **13** 315
Stauffer D and Oliveira P C M 2002 *Eur. Phys. J.* B **30** 587
[52] Stauffer D 2002 *Int. J. Mod. Phys.* C **13** 975
[53] Krapivsky P L and Redner S 2003 *Phys. Rev. Lett.* **90** 238701
[54] Mobilia M 2003 *Phys. Rev. Lett.* **91** 028701
[55] Tessone C, Amengual P, Toral R, Wio H S and San Miguel M 2004 *Eur. Phys. J.* B **25** 403
[56] Laguna M F, Risau Gusman S, Abramson G, Goncalves S and Iglesias J R 2005 *Physica* A **351** 580
[57] Schneider J J 2004 *Int. J. Mod. Phys.* C **15** 659
Schneider J J and Hirtreiter Ch 2005 *Physica* A **353** 539
Schneider J J and Hirtreiter Ch 2005 *Int. J. Mod. Phys.* C **16** 157
[58] Tessone C, Toral R, Amengual P, Wio H S and San Miguel M 2004 *Eur. Phys. J.* B **39** 535
[59] Toral R and Tessone C 2006 *Commun. Comput. Phys.* **2** 177
[60] de la Lama M S, Szendro I G, Iglesias J R and Wio H S 2006 *Eur. Phys. J.* B **51** 435
[61] He M, Sun Q and Wang H 2004 *Int. J. Mod. Phys.* C **15** 767
[62] Landau D P and Binder K 2000 *A Guide to Monte Carlo Simulations in Statistical Physics* (Cambridge: Cambridge University Press)
[63] Sides S W, Rikvold P A and Novotny M A 1998 *Phys. Rev. Lett.* **81** 834
[64] Bordogna C M and Albano E V 2006 *Int. J. Mod. Phys.* C **17** 409